GENERALIZED SOLUTIONS OF NONSTEADY HEAT AND MASS TRANSFER

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We have solved a system of differential equations for the simultaneous transport of moisture and heat in the case of a plate, a cylinder, and a sphere, with boundary conditions of the first, second, and third kinds. We also consider the interrelated transfer of heat and moisture during the period of the declining drying rate.

Fourier's classic work [1] provided the basis for the analytical theory of transport, which is now one of the most widely developed branches of mathematical physics. Such a large number of papers and books have appeared that it is probably impossible to enumerate the entire literature devoted to questions of the transfer of heat or mass in a solid. The accumulation of such substantial information demands new forms of storage and generalization.

Gol'dfarb [2] was the first to suggest combined solutions for a plate, a cylinder, and a sphere, and then a new finite integral transformation which is a consolidation of the Fourier and Hankel transforms [3].

An analogous hypergeometric transformation was developed in [4]. This transformation was used in [5] to achieve a generalized solution for the equation of heat conduction, with consideration given to the internal sources of heat for extremely general initial and boundary conditions.

In this article we have derived a generalized solution for a system of differential equations [6] for moisture and heat transport:

$$\frac{\partial\theta_1(\xi,\ \mathrm{Fo})}{\partial\,\mathrm{Fo}}=\frac{\partial^2\theta_1(\xi,\ \mathrm{Fo})}{\partial\xi^2}+\frac{\Gamma}{\xi}\ \frac{\partial\theta_1(\xi,\ \mathrm{Fo})}{\partial\xi}-\mathrm{Ko}^*\ \frac{\partial\theta_2(\xi,\ \mathrm{Fo})}{\partial\,\mathrm{Fo}}, \tag{1}$$

$$\frac{\partial\theta_2(\xi,\ \mathrm{Fo})}{\partial\,\mathrm{Fo}}=\mathrm{Lu}\left[\frac{\partial^2\theta_2(\xi,\ \mathrm{Fo})}{\partial\xi^2}+\frac{\Gamma}{\xi}\ \frac{\partial\theta_2(\xi,\ \mathrm{Fo})}{\partial\xi}\right]-\mathrm{Lu\ Pn}\left[\frac{\partial^2\theta_1(\xi,\ \mathrm{Fo})}{\partial\xi^2}+\frac{\Gamma}{\xi}\ \frac{\partial\theta_1(\xi,\ \mathrm{Fo})}{\partial\xi}\right]. \tag{2}$$

The initial values of the potentials are assumed to be specified functions of the space coordinate

$$\theta_k(\xi,\ 0)=f_k(\xi)\qquad (k=1,\ 2). \tag{3}$$

Moreover, in view of the symmetry, for a plate we have

$$\frac{\partial\theta_k(0,\ \mathrm{Fo})}{\partial\xi}=0\qquad (k=1,\ 2). \tag{4}$$

After initially applying the finite transformation [5]

$$\{\theta_k(\mu,\ \mathrm{Fo})\}_\Gamma=\int_0^1\xi^\Gamma\Phi_\Gamma(\mu\xi)\,\theta_k(\xi,\ \mathrm{Fo})\,d\xi, \tag{5}$$

and then the Laplace transform

$$\{\bar\theta_k(\mu,s)\}_\Gamma=\int_0^\infty\{\theta_k(\mu,\ \mathrm{Fo})\}_\Gamma\exp(-s\,\mathrm{Fo})\,d\,\mathrm{Fo}, \tag{6}$$

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TABLE 1. Values of $\mathrm{Lu}\vartheta_i^2$

Lu	$\mathrm{Lu}\vartheta_i^2$	Ko*Pn					
		0,1	0,2	0,4	0,6	0,8	1,0
0,01	$\mathrm{Lu}\vartheta_1^2$	0,00999	0,00998	0,00996	0,00994	0,00992	0,00990
	$\mathrm{Lu}\vartheta_2^2$	1,00101	1,00202	1,00404	1,00606	1,00808	1,01010
0,05	$\mathrm{Lu}\vartheta_1^2$	0,04974	0,04948	0,04897	0,04847	0,04798	0,04751
	$\mathrm{Lu}\vartheta_2^2$	1,00526	1,01052	1,02103	1,03153	1,04202	1,05249
0,1	$\mathrm{Lu}\vartheta_1^2$	0,09890	0,09783	0,09576	0,09379	0,09190	0,09010
	$\mathrm{Lu}\vartheta_2^2$	1,01110	1,02217	1,04424	1,06621	1,08810	1,10990
0,2	$\mathrm{Lu}\vartheta_1^2$	0,19515	0,19058	0,18218	0,17461	0,16775	0,16148
	$\mathrm{Lu}\vartheta_2^2$	1,02485	1,04942	1,09782	1,14539	1,19225	1,23852
0,4	$\mathrm{Lu}\vartheta_1^2$	0,37591	0,35581	0,32349	0,29808	0,27725	0,25969
	$\mathrm{Lu}\vartheta_2^2$	1,06409	1,12419	1,23651	1,34192	1,44275	1,54031
0,6	$\mathrm{Lu}\vartheta_1^2$	0,53184	0,48637	0,42361	0,37967	0,34603	0,31898
	$\mathrm{Lu}\vartheta_2^2$	1,12816	1,23363	1,41639	1,58033	1,73397	1,88102
0,8	$\mathrm{Lu}\vartheta_1^2$	0,65086	0,57950	0,49114	0,43318	0,39030	0,35660
	$\mathrm{Lu}\vartheta_2^2$	1,22914	1,38050	1,62886	1,84682	2,04970	2,24340
1,0	$\mathrm{Lu}\vartheta_1^2$	0,72984	0,64174	0,53668	0,46934	0,42020	0,38197
	$\mathrm{Lu}\vartheta_2^2$	1,37016	1,55826	1,86332	2,13066	2,37980	2,61803

to system (1)-(2), in conjunction with (3)-(4), we obtain an algebraic system of equations which is solved for the images $\{\bar{\theta}_k(\mu,\ s)\}_\Gamma$. We then find the originals of these potentials, initially for the parameter s, and then, from Fo, with the aid of the inversion formula from [5], we find

$$\theta_k(\xi,\ \mathrm{Fo}) = \sum_{n=1}^{\infty} \frac{2\Phi_\Gamma(\mu_n\xi)}{\Phi_\Gamma^2(\mu_n) + V_\Gamma^2(\mu_n) + \dfrac{1-\Gamma}{\mu_n}\Phi_\Gamma(\mu_n)V_\Gamma(\mu_n)} \{\theta_k(\mu_n,\ \mathrm{Fo})\}_\Gamma . \tag{7}$$

The final solution is in the form

$$\theta_1(\xi,\ \mathrm{Fo}) = \sum_{k=1}^{2}\sum_{i=1}^{2} A_{ki}C_{ki}, \tag{8}$$

$$\theta_2(\xi,\ \mathrm{Fo}) = \sum_{k=1}^{2}\sum_{i=1}^{2} B_{ki}C_{ki}, \tag{9}$$

where

$$A_{ki} = (-1)^i \frac{(k-1)\,\mathrm{Ko}^* - (k-2)(1-\vartheta_i^2)}{\vartheta_1^2 - \vartheta_2^2}; \tag{10}$$

$$B_{ki} = (-1)^i \frac{(k-1)\left(\dfrac{1}{\mathrm{Lu}\,\vartheta_i^2} - 1\right) - (k-2)\,\mathrm{Pn}}{\vartheta_1^2 - \vartheta_2^2}; \tag{11}$$

$$\vartheta_i^2 = \frac{1}{2}\left[\left(1 + \mathrm{Ko}^*\,\mathrm{Pn} + \frac{1}{\mathrm{Lu}}\right) + (-1)^i \sqrt{\left(1 + \mathrm{Ko}^*\,\mathrm{Pn} + \frac{1}{\mathrm{Lu}}\right)^2 - \frac{4}{\mathrm{Lu}}}\right]; \tag{12}$$

$$C_{ki} = \sum_{n=1}^{\infty} \frac{2\Phi_\Gamma(\mu_n\xi)}{\Phi_\Gamma^2(\mu_n) + V_\Gamma^2(\mu_n) + \dfrac{1-\Gamma}{\mu_n}\Phi_\Gamma(\mu_n)V_\Gamma(\mu_n)} \exp[-\mu_n^2\vartheta_i^2\,\mathrm{Lu\,Fo}]\left\{\int_0^1 \xi^\Gamma \Phi_\Gamma(\mu_n\xi) f_k(\xi)\,d\xi\right.$$
$$\left. + \int_0^{\mathrm{Fo}} \left[\Phi_\Gamma(\mu_n)\frac{\partial\theta(1,\ \mathrm{Fo}^*)}{\partial\xi} + \mu_n V_\Gamma(\mu_n)\,\theta(1,\ \mathrm{Fo}^*)\right] \exp[\mu_n^2\vartheta_i^2\,\mathrm{Lu\,Fo}^*]\, d(\vartheta_i^2\,\mathrm{Lu\,Fo}^*)\right\}. \tag{13}$$

TABLE 2. Value of the Coefficient $A_{11} = B_{22}$

Lu	Ko*Pn					
	0,1	0,2	0,4	0,6	0,8	1,0
0,01	0,0000	0,0000	0,0000	0,0001	0,0001	0,0001
0,05	0,0003	0,0005	0,0011	0,0016	0,0020	0,0025
0,1	0,0012	0,0023	0,0045	0,0064	0,0081	0,0097
0,2	0,0058	0,0110	0,0195	0,0262	0,0315	0,0358
0,4	0,0350	0,0575	0,0838	0,0976	0,1053	0,1096
0,6	0,1143	0,1521	0,1777	0,1835	0,1830	0,1799
0,8	0,2579	0,2753	0,2715	0,2595	0,2469	0,2350
1,0	0,4219	0,3909	0,3492	0,3194	0,2959	0,2764

If we assume in (13) that $\mathrm{Fo}_i = \vartheta_i^2 \,\mathrm{Lu\,Fo}$, the expression for C_{ki} coincides completely with the generalized solution of the heat-conduction equation derived in [5]. This enables us to use (13) to calculate the particular solutions of "pure" heat conduction.

Table 1 shows the values of $\mathrm{Lu}\,\vartheta_i^2$ $(i = 1, 2)$ in the interval of variation for the Luikov number from 0.01 to 1 and for Ko*Pn from 0.1 to 1. The values of A_{11} and B_{22} for the same values of the Luikov number Lu and Ko*Pn are given in Table 2. The values of B_{11}, for a change in Lu from 0.01 to 1.0, and for a change in Ko*Pn and Pn from 0.1 to 1.0, are given in Table 3. The values of A_{21} for the same values of Lu, Ko*Pn, and Pn are given in Table 4. The remaining coefficients are easily derived from the same tables, since $A_{22} = -A_{21}$, $B_{12} = -B_{11}$, $B_{21} = 1 - B_{22}$, and $A_{12} = 1 - A_{11}$.

Under boundary conditions of the first kind the values of the dimensionless potentials at the surface of the body are specified:

$$\theta_k(1,\ \mathrm{Fo}) = \varphi_k(\mathrm{Fo}) \qquad (k = 1,\ 2). \tag{14}$$

For this case we must assume that the values of μ_n are the roots of the characteristic equation

$$\Phi_\Gamma(\mu) = 0. \tag{15}$$

With consideration of (14) and (15), we find that formula (13) assumes the form

$$C_{ki} = \sum_{n=1}^{\infty} \frac{2\Phi_\Gamma(\mu_n\xi)}{V_\Gamma^2(\mu_n)} \exp[-\mu_n^2 \vartheta_i^2 \,\mathrm{Lu\,Fo}] \left\{ \int_0^1 \xi^\Gamma \Phi_\Gamma(\mu_n\xi) f_k(\xi)\, d\xi \right.$$
$$\left. + \mu_n V_\Gamma(\mu_n) \int_0^{\mathrm{Fo}} \varphi_k(\mathrm{Fo}^*) \exp[\mu_n^2 \vartheta_i^2 \,\mathrm{Lu\,Fo}^*]\, d(\vartheta_i^2 \,\mathrm{Lu\,Fo}^*) \right\}. \tag{16}$$

The derivatives of the dimensionless transport potentials at the surface of the body are specified under boundary conditions of the second kind:

$$\frac{\partial\theta_k(1,\ \mathrm{Fo})}{\partial\xi} = \psi_k(\mathrm{Fo}) \qquad (k = 1,\ 2). \tag{17}$$

For this case we must assume that μ_n denotes the roots of the characteristic equation

$$V_\Gamma(\mu_n) = 0. \tag{18}$$

With consideration of (17) and (18) in addition to the fact that $\mu = 0$ is also a root of (18), we find that (13) assumes the form

$$C_{ki} = (\Gamma + 1)\left[\int_0^1 \xi^\Gamma f_k(\xi)\, d\xi + \int_0^{\mathrm{Fo}} \psi_k(\mathrm{Fo}^*)\, d(\vartheta_i^2 \,\mathrm{Lu\,Fo}^*) \right]$$
$$+ \sum_{n=1}^{2} \frac{2\Phi_\Gamma(\mu_n\xi)}{\Phi_\Gamma^2(\mu_n)} \exp[-\mu_n^2 \vartheta_i^2 \,\mathrm{Lu\,Fo}] \left\{ \int_0^1 \xi^\Gamma \Phi_\Gamma(\mu_n\xi) f_k(\xi)\, d\xi \right.$$
$$\left. + \Phi_\Gamma(\mu_n) \int_0^{\mathrm{Fo}} \psi_k(\mathrm{Fo}^*) \exp[\mu_n^2 \vartheta_i^2 \,\mathrm{Lu\,Fo}^*]\, d(\vartheta_i^2 \,\mathrm{Lu\,Fo}) \right\}. \tag{19}$$

TABLE 3. Value of the Coefficient B_{11}

Lu	Pn	Ko*Pn 0,1	0,2	0,4	0,6	0,8	1,0
0,01	0,1	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010
	0,2	0,0020	0,0020	0,0020	0,0020	0,0020	0,0020
	0,4	0,0040	0,0040	0,0040	0,0040	0,0040	0,0040
	0,6	0,0060	0,0060	0,0060	0,0060	0,0060	0,0060
	0,8	0,0081	0,0081	0,0080	0,0080	0,0080	0,0080
	1,0	0,0101	0,0101	0,0101	0,0100	0,0100	0,0100
0,05	0,1	0,0052	0,0052	0,0051	0,0051	0,0050	0,0050
	0,2	0,0105	0,0104	0,0103	0,0102	0,0101	0,0100
	0,4	0,0209	0,0208	0,0206	0,0203	0,0201	0,0199
	0,6	0,0314	0,0312	0,0309	0,0305	0,0302	0,0298
	0,8	0,0419	0,0416	0,0411	0,0407	0,0402	0,0398
	1,0	0,0523	0,0520	0,0514	0,0509	0,0503	0,0498
0,1	0,1	0,0110	0,0108	0,0105	0,0103	0,0100	0,0098
	0,2	0,0219	0,0216	0,0211	0,0206	0,0201	0,0196
	0,4	0,0438	0,0433	0,0422	0,0411	0,0402	0,0392
	0,6	0,0658	0,0649	0,0633	0,0617	0,0602	0,0588
	0,8	0,0877	0,0865	0,0843	0,0823	0,0803	0,0784
	1,0	0,1096	0,1082	0,1054	0,1004	0,1004	0,0981
0,2	0,1	0,0241	0,0233	0,0218	0,0206	0,0195	0,0186
	0,2	0,0482	0,0466	0,0437	0,0412	0,0390	0,0371
	0,4	0,0964	0,0931	0,0874	0,0824	0,0781	0,0743
	0,6	0,1446	0,1397	0,1311	0,1236	0,1171	0,1114
	0,8	0,1928	0,1863	0,1747	0,1648	0,1562	0,1486
	1,0	0,2410	0,2329	0,2184	0,2060	0,1952	0,1857
0,4	0,1	0,0581	0,0521	0,0438	0,0383	0,0343	0,0312
	0,2	0,1162	0,1041	0,0876	0,0766	0,0686	0,0625
	0,4	0,2325	0,2082	0,1752	0,1533	0,1373	0,1249
	0,6	0,3487	0,3123	0,2629	0,2299	0,2059	0,1874
	0,8	0,4650	0,4165	0,3505	0,3066	0,2746	0,2499
	1,0	0,5812	0,5206	0,4381	0,3832	0,3432	0,3123
0,6	0,1	0,1006	0,0803	0,0604	0,0500	0,0432	0,0384
	0,2	0,2012	0,1606	0,1209	0,0999	0,0865	0,0768
	0,4	0,4025	0,3212	0,2417	0,1999	0,1729	0,1536
	0,6	0,6037	0,4818	0,3626	0,2998	0,2594	0,2305
	0,8	0,8049	0,6423	0,4835	0,3998	0,3458	0,3073
	1,0	1,0062	0,8029	0,6044	0,4997	0,4323	0,3841
0,8	0,1	0,1383	0,0999	0,0703	0,0566	0,0482	0,0424
	0,2	0,2767	0,1998	0,1406	0,1132	0,0964	0,0848
	0,4	0,5534	0,3995	0,2813	0,2264	0,1928	0,1696
	0,6	0,8301	0,5992	0,4219	0,3395	0,2893	0,2544
	0,8	1,1067	0,7990	0,5625	0,4527	0,3857	0,3392
	1,0	1,3834	0,9987	0,7032	0,5659	0,4821	0,4240
1,0	0,1	0,1562	0,1091	0,0754	0,0602	0,0510	0,0447
	0,2	0,3123	0,2182	0,1507	0,1204	0,1021	0,0894
	0,4	0,6247	0,4364	0,3015	0,2408	0,2041	0,1789
	0,6	0,9370	0,6546	0,4523	0,3612	0,3062	0,2683
	0,8	1,2494	0,8729	0,6030	0,4815	0,4082	0,3578
	1,0	1,5617	1,0911	0,7538	0,6019	0,5103	0,4472

TABLE 4. Values of the Coefficient A_{21}

Lu	Pn	Ko*Pn					
		0.1	0,2	0,4	0,6	0,8	1,0
0,01	0,1	0,0101	0,0202	0,0402	0,0602	0,0801	0,1000
	0,2	0,0050	0,0101	0,0201	0,0301	0,0401	0,0500
	0,4	0,0025	0,0050	0,0101	0,0151	0,0200	0,0250
	0,6	0,0017	0,0034	0,0067	0,0100	0,0134	0,0167
	0,8	0,0013	0,0025	0,0050	0,0075	0,0100	0,0125
	1,0	0,0010	0,0020	0,0040	0,0060	0,0080	0,0100
0,05	0,1	0,0523	0,1040	0,2057	0,3052	0,4024	0,4975
	0,2	0,0262	0,0520	0,1029	0,1526	0,2012	0,2488
	0,4	0,0131	0,0260	0,0514	0,0763	0,1006	0,1244
	0,6	0,0087	0,0173	0,0343	0,0509	0,0671	0,0829
	0,8	0,0065	0,0130	0,0257	0,0381	0,0503	0,0622
	1,0	0,0052	0,0104	0,0206	0,0305	0,0402	0,0498
0,1	0,1	0,1096	0,2164	0,4217	0,6170	0,8031	0,9806
	0,2	0,0541	0,1082	0,2109	0,3085	0,4015	0,4903
	0,4	0,0274	0,0541	0,1054	0,1542	0,2008	0,2451
	0,6	0,0183	0,0361	0,0703	0,1028	0,1338	0,1634
	0,8	0,0137	0,0270	0,0527	0,0771	0,1004	0,1226
	1,0	0,0110	0,0216	0,0422	0,0617	0,0803	0,0981
0,2	0,1	0,2411	0,4657	0,8737	1,2361	1,5617	1,8570
	0,2	0,1205	0,2329	0,4368	0,6181	0,7809	0,9285
	0,4	0,0603	0,1164	0,2184	0,3090	0,3904	0,4642
	0,6	0,0402	0,0776	0,1456	0,2060	0,2603	0,3095
	0,8	0,0301	0,0582	0,1092	0,1545	0,1952	0,2321
	1,0	0,0241	0,0466	0,0874	0,1236	0,1562	0,1857
0,4	0,1	0,5812	1,0412	1,7524	2,2992	2,7456	3,1235
	0,2	0,2906	0,5206	0,8762	1,1496	1,3728	1,5617
	0,4	0,1453	0,2603	0,4381	0,5748	0,6864	0,7809
	0,6	0,0969	0,1735	0,2921	0,3832	0,4576	0,5206
	0,8	0,0726	0,1301	0,2190	0,2874	0,3432	0,3904
	1,0	0,0581	0,1041	0,1752	0,2299	0,2746	0,3123
0,6	0,1	1,0062	1,6059	2,4175	2,9983	3,4583	3,8411
	0,2	0,5031	0,8029	1,2087	1,4992	1,7292	1,9206
	0,4	0,2515	0,4015	0,6044	0,7496	0,8646	0,9603
	0,6	0,1677	0,2676	0,4029	0,4997	0,5764	0,6402
	0,8	0,1258	0,2007	0,3022	0,3748	0,4323	0,4801
	1,0	0,1006	0,1606	0,2417	0,2998	0,3458	0,3841
0,8	0,1	1,3834	1,9975	2,8126	3,3955	3,8568	4,2400
	0,2	0,6917	0,9988	1,4063	1,6977	1,9284	2,1200
	0,4	0,3459	0,4994	0,7032	0,8489	0,9642	1,0600
	0,6	0,2306	0,3329	0,4688	0,5659	0,6428	0,7067
	0,8	0,1729	0,2497	0,3516	0,4244	0,4821	0,5300
	1,0	0,1383	0,1998	0,2813	0,3395	0,3857	0,4240
1,0	0,1	1,5617	2,1822	3,0151	3,6116	4,0825	4,4721
	0,2	0,7809	1,0911	1,5076	1,8058	2,0412	2,2361
	0,4	0,3904	0,5455	0,7538	0,9029	1,0206	1,1180
	0,6	0,2603	0,3637	0,5025	0,6019	0,6804	0,7454
	0,8	0,1952	0,2728	0,3769	0,4514	0,5103	0,5590
	1,0	0,1562	0,2182	0,3015	0,3612	0,4082	0,4472

The boundary conditions at the surface of the body, with the dimensionless flows of heat and matter specified as functions of time, assume the form [6]:

$$\frac{\partial\theta_1(1,\ \mathrm{Fo})}{\partial\xi} = \mathrm{Ki}_1(\mathrm{Fo}) - (1-\varepsilon)\,\mathrm{Lu}\,\mathrm{Ko}\,\mathrm{Ki}_2(\mathrm{Fo}), \tag{20}$$

$$\frac{\partial\theta_2(1,\ \mathrm{Fo})}{\partial\xi} = \mathrm{Pn}\,\frac{\partial\theta_1(1,\ \mathrm{Fo})}{\partial\xi} + \mathrm{Ki}_2(\mathrm{Fo}). \tag{21}$$

From these relationships we can express the function $\psi_k(\mathrm{Fo})$ in terms of the flows $\mathrm{Ki}_k(\mathrm{Fo})$ as follows:

$$\psi_1(\mathrm{Fo}) = \mathrm{Ki}_1(\mathrm{Fo}) - (1-\varepsilon)\,\mathrm{Lu}\,\mathrm{Ko}\,\mathrm{Ki}_2(\mathrm{Fo}), \tag{22}$$

$$\psi_2(\mathrm{Fo}) = \mathrm{Pn}\,\mathrm{Ki}_1(\mathrm{Fo}) + [1-\mathrm{Pn}(1-\varepsilon)\,\mathrm{Lu}\,\mathrm{Ko}]\,\mathrm{Ki}_2(\mathrm{Fo}). \tag{23}$$

For the quantitative calculations Luikov [7] replaced the drying-rate curve during the period of a declining rate by the straight line

$$-\frac{\partial\bar{u}}{\partial\tau} = \varkappa N\,(\bar{u} - \bar{u}_p). \tag{24}$$

Based on numerous tests on the drying of the most diverse materials, in reference [8] the following empirical formula has been derived: $\varkappa = 1.8/\bar{u}_0$.

Equation (24) in dimensionless form will be

$$\mathrm{Ki}_2(\mathrm{Fo}) = K\left[\frac{1}{\Gamma+1} - \int\limits_0^1 \xi^{\Gamma}\theta_2(\xi,\ \mathrm{Fo})\,d\xi\right]. \tag{25}$$

The relationship between the intensity of heat and moisture transport, which has the form

$$\mathrm{Ki}_1(\mathrm{Fo}) = \mathrm{Lu}\,\mathrm{Ko}\,(1+\mathrm{Rb})\,\mathrm{Ki}_2(\mathrm{Fo}) \tag{26}$$

has been established on the basis of the law of the conservation of energy in [9].

The Rebinder number $\mathrm{Rb} = \beta(c/\rho)$ for a regular drying regime is a constant, because for this regime $\beta = \mathrm{const}$, as had been demonstrated experimentally by Luikov as far back as 1934 [7].

Substitution of (25) and (26) into (22) and (23) yields

$$\psi_k(\mathrm{Fo}) = M_k\left\{\frac{1}{\Gamma+1} - \int\limits_0^1 \xi^{\Gamma}\theta_2(\xi,\ \mathrm{Fo})\,d\xi\right\} \qquad (k=1,\ 2), \tag{27}$$

where

$$M_1 = K\,\mathrm{Lu}\,\mathrm{Ko}\,(\varepsilon+\mathrm{Rb}), \tag{28}$$

$$M_2 = K\,\mathrm{Lu}\,\mathrm{Ko}\,(\varepsilon+\mathrm{Rb}) + K. \tag{29}$$

Differentiating expression (27) for Fo

$$\frac{d\psi_k(\mathrm{Fo})}{d\,\mathrm{Fo}} = -M_k\int\limits_0^1 \xi^{\Gamma}\,\frac{\partial\theta_2(\xi,\ \mathrm{Fo})}{\partial\,\mathrm{Fo}}\,d\xi. \tag{30}$$

After substitution of (2) into (30) and following integration, with consideration of (3) and (17) we find

$$\frac{d\psi_k(\mathrm{Fo})}{d\,\mathrm{Fo}} = M_k\,\mathrm{Lu}\,[\mathrm{Pn}\,\psi_1(\mathrm{Fo}) - \psi_2(\mathrm{Fo})] \qquad (k=1,\ 2). \tag{31}$$

Applying the Laplace transform to system (31), we find an algebraic system of equations from which we determine

$$\bar{\psi}_k(s) = \frac{M_k}{s+\mathrm{Lu}\,K}\left\{\frac{1}{\Gamma+1} - \int\limits_0^1 \xi^{\Gamma} f_2(\xi)\,d\xi\right\}. \tag{32}$$

Turning to the original of the parameter s, we find

$$\psi_k(\mathrm{Fo}) = M_k\left\{\frac{1}{\Gamma+1} - \int_0^1 \xi^\Gamma f_2(\xi)\,d\xi\right\}\exp(-K\,\mathrm{Lu}\,\mathrm{Fo}). \tag{33}$$

Substitution of (33) into (19) yields

$$C_{ki} = (\Gamma+1)\int_0^1 \xi^\Gamma f_k(\xi)\,d\xi + \sum_{n=1}^{\infty}\frac{2\Phi_\Gamma(\mu_n\xi)}{\Phi_\Gamma^2(\mu_n)}\exp(-\mu_n^2\vartheta_i^2\,\mathrm{Lu}\,\mathrm{Fo})\int_0^1 \xi^\Gamma\Phi_\Gamma(\mu_n\xi)f_k(\xi)\,d\xi$$

$$+\frac{M_k}{\Gamma+1}\left[1-(\Gamma+1)\int_0^1 \xi^\Gamma f_2(\xi)\,d\xi\right]\left\{(\Gamma+1)\frac{\vartheta_i^2}{K} - \frac{\Phi_\Gamma\left(\xi\sqrt{\frac{K}{\vartheta_i^2}}\right)}{\sqrt{\frac{K}{\vartheta_i^2}}\,V_\Gamma\left(\sqrt{\frac{K}{\vartheta_i^2}}\right)}\exp(-K\,\mathrm{Lu}\,\mathrm{Fo})\right.$$

$$\left.-\sum_{n=1}^{\infty}\frac{2\Phi_\Gamma(\mu_n\xi)}{\left(\mu_n^2-\frac{K}{\vartheta_i^2}\right)\Phi_\Gamma(\mu_n)}\exp(-\mu_n^2\vartheta_i^2\,\mathrm{Lu}\,\mathrm{Fo})\right\}. \tag{34}$$

The determination of the temperature–moisture distribution during a period of a declining drying rate is thus reduced to the solution of system of heat and mass transport equations for boundary conditions of the second kind, the only difference being that we must take (34) instead of (19).

We specify the linear combination of the dimensionless potentials and their derivatives at the material surface under boundary conditions of the third kind as a function of time:

$$\theta_k(1,\ \mathrm{Fo}) + \frac{1}{\mathrm{Bi}_k}\frac{\partial\theta_k(1,\ \mathrm{Fo})}{\partial\xi} = \varphi_k(\mathrm{Fo}) \qquad (k=1,\ 2). \tag{35}$$

This definition of the boundary conditions differs from that adopted in [6]. It is equivalent to assuming Pn = 0 in the boundary conditions of [6], which is satisfied when there is no thermal-gradient transport of matter in the material.

For this case we have to assume that $\mu_{n,k}$ denotes the roots of the characteristic equation

$$\frac{\Phi_\Gamma(\mu)}{V_\Gamma(\mu)} = \frac{\mu}{\mathrm{Bi}_k} \qquad (k=1,\ 2). \tag{36}$$

In conjunction with (35) and (36), formula (13) assumes the form

$$C_{ki} = \sum_{n=1}^{\infty}\frac{2\Phi_\Gamma(\mu_{n,k}\xi)}{V_\Gamma^2(\mu_{n,k})}\,\frac{\mathrm{Bi}_k^2}{\mu_{n,k}^2+\mathrm{Bi}_k^2+(1-\Gamma)\mathrm{Bi}_k}\exp(-\mu_n^2\vartheta_i^2\,\mathrm{Lu}\,\mathrm{Fo})\left\{\int_0^1 \xi^\Gamma\Phi_\Gamma(\mu_n\xi)f_k(\xi)\,d\xi\right.$$

$$\left.+\mu_{n,k}V_\Gamma(\mu_{n,k})\int_0^{\mathrm{Fo}}\varphi_k(\mathrm{Fo}^*)\exp(\mu_{n,k}^2\vartheta_i^2\,\mathrm{Lu}\,\mathrm{Fo}^*)\,d(\vartheta_i^2\,\mathrm{Lu}\,\mathrm{Fo}^*)\right\}. \tag{37}$$

We can easily derive all types of particular solutions from these. To achieve specific solutions for a plate, a cylinder, and a sphere, we have to use the relationships

$$\Phi_0(x) = \cos x,\quad \Phi_1(x) = I_0(x),\quad \Phi_2(x) = \frac{\sin x}{x},$$
$$V_0(x) = \sin x,\quad V_1(x) = I_1(x),\quad V_2(x) = \frac{\sin x - x\cos x}{x^2}. \tag{38}$$

NOTATION

ξ is a dimensionless coordinate;
Fo is the Fourier number;

$\theta_1(\xi, \mathrm{Fo})$	is the dimensionless temperature of the material;
$\theta_2(\xi, \mathrm{Fo})$	is the dimensionless potential of mass transport;
Ko*	is the modified Kosovich number;
Lu	is the Luikov number;
Pn	is the Posnov number;
Γ	is the form factor of the material, equal to 0, 1, and 2, respectively, for a plate, a cylinder, and a sphere;
$\mathrm{Ki}_1(\mathrm{Fo})$	is the dimensionless heat flow;
$\mathrm{Ki}_2(\mathrm{Fo})$	is the dimensionless mass flow;
$K = \varkappa NR/a_m$;	
Rb	is the Rebinder number;
Bi	is the Biot number.

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